

Lab Manual

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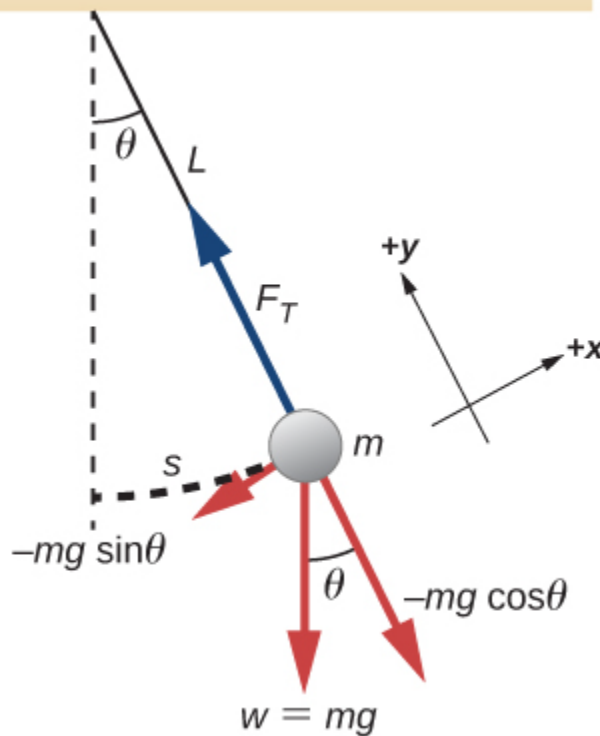
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PART ONE: DETERMINING THE ACCELERATION DUE TO GRAVITY, “g”, FROM THE PERIOD OF A PENDULUM

INTRODUCTION

A **simple pendulum** is defined to have a point mass, also known as the pendulum bob, which is suspended from a string of length L with negligible mass. Here, the only forces acting on the bob are the force of gravity (i.e., the weight of the bob) and tension from the string. The mass of the string is assumed to be negligible as compared to the mass of the bob.

Figure



simple pendulum has a small-diameter bob and a string that has a very small mass but is strong enough not to stretch appreciably. The linear displacement from equilibrium is s , the length of the arc. Also shown are the forces on the bob, which result in a net force of $-mg \sin \theta$ toward the equilibrium position—that is, a restoring force.

When displaced to an initial angle and released, the pendulum will swing back and forth with periodic motion. By applying Newton's second law for rotational systems, the equation of motion for the pendulum may be obtained

$$\tau = I\alpha \Rightarrow -mg \sin\theta L = mL^2 \frac{d^2\theta}{dt^2},$$

and rearranged as

$$\frac{d^2\theta}{dt^2} + \frac{g}{L} \sin\theta = 0.$$

If the amplitude of angular displacement is small enough that the small angle approximation ($\sin\theta \approx \theta$) holds true, then the equation of motion reduces to the equation of simple harmonic motion

$$\frac{d^2\theta}{dt^2} + \frac{g}{L} \theta = 0.$$

The simple harmonic solution is

$$\theta(t) = \theta_0 \cos(\omega t + \phi)$$

with $\omega = \sqrt{g/L}$ being the natural frequency of the motion.

With the assumption of small angles, the frequency and period of the pendulum are independent of the initial angular displacement amplitude. All simple pendulums should have the same period regardless of their initial angle (and regardless of their masses).

The period of a pendulum depends on the acceleration due to gravity, “g” as well as other factors.

The period for a simple pendulum does not depend on the mass or the initial angular displacement, but depends only on the length L of the string and the value of the gravitational field strength g , according to

$$T = 2\pi\sqrt{\frac{L}{g}}$$

If the formula for the period of a pendulum can be rearranged to express “g” as a slope, then measurements can be taken and “g” can be estimated as the slope of a line. Solving for “g” in the formula for the period of the pendulum gives

$$g = \frac{4\pi^2 L}{T^2}$$

This is a slope with $4\pi^2 L$ on the vertical axis and T^2 on the horizontal axis.

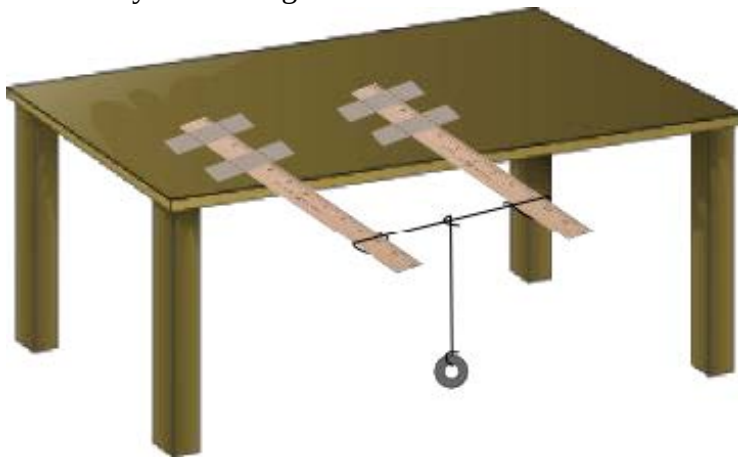
EQUIPMENT

1.5 meters of string
5 large washers (see materials list for a picture)
1 clock
1 meter stick

PROCEDURE

Investigating Period and Mass

1. Construct a pendulum by tying a large washer to the string.
 - a. If you're using duct tape, tape down two rulers to the table about 6 inches apart so that several inches stick out beyond the edge of the table as shown in the diagram below. If you're using masking tape instead, tape two rulers to opposite sides of the Tupperware container at the same height so that several inches stick out beyond the edge of the container. Place the box-ruler setup on the edge of the table such that the rulers stick out beyond the edge of the table.



2. The period of the pendulum is the time to complete a cycle (back and forth). The period of the pendulum (seconds/cycle) can be obtained by counting the number of seconds needed for the pendulum to make 10 complete cycles and dividing by 10.
3. For five different pendulum masses, find the period of the pendulum. Repeat the experiment several times for each mass, averaging your data for each mass.
4. Plot a graph of T versus mass.

Investigating Period and Length

1. Construct a pendulum by tying two large washers to the string.

2. Construct a Data Table as follows:

DATA TABLE			
Length (m)	Period (sec/cycle)	$4\pi^2 L$	T^2
_____	_____	_____	_____
_____	_____	_____	_____

3. For five different pendulum lengths, obtain and record the period of the pendulum. (you have to repeat the experiment several times for each length, averaging your data for each length). determine the best estimate of the error of this measurement.

4. Plot a graph of $4\pi^2 L$ on the vertical axis and T^2 by hand to confirm that the plot looks approximately like a straight line.

5. Draw the line-of-best-fit for the plot of $4\pi^2 L$ versus T^2 you made in step #4 and find the slope of the line as you did in a previous lab (use points on the line not data points in finding the slope).

RESULTS

1. What did you find for the slope of $4\pi^2 L$ versus T^2 (i.e., what was your estimate for the acceleration due to gravity, “g”)?

2. Draw a graph of the period of the pendulum vs. length. Is it linear? Given the formula for the period of the pendulum would you expect the relationship between period and length to be linear?

3. Calculate the percent error of your experimental result.

4. What are the top two sources of error in this experiment?

PART TWO: ESTIMATING THE SPRING CONSTANT OF A RUBBER BAND USING LINEAR REGRESSION

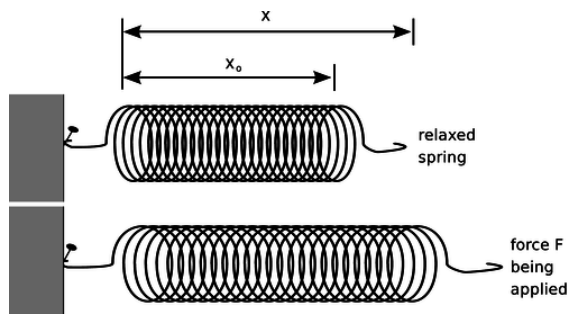
INTRODUCTION

The displacement of an elastic object (e.g., spring or rubber band) increases in proportion to the applied force (for small displacements). This was discovered by Robert Hooke in the 1700's and is called Hooke's Law.

Statement of Hooke's Law. The physical setup in which a mass is connected to a spring is a rather ubiquitous one; for example, molecular structures are often so modelled. We don't consider gravity in this situation, so there are only two constants of relevance to us. The first is the so-called **spring constant**, which measures the "stiffness" of the spring (the larger the constant, the greater the stiffness). The second constant is the (inertial) mass of the object connected to the spring. **Hooke's Law** says that the force exerted on the object by the spring is proportion to the displacement away from equilibrium (upstretched spring). Since Newton's Second Law says that the net force on an object is the object's mass times the acceleration resulting from the force(s), and since acceleration is represented by the second derivative of position with respect to time, we conclude that Hooke's Law takes the form of a **differential equation**:

$$m \frac{d^2 x}{dt^2} = -kx$$

where k is the spring constant.



In this experiment, you will confirm Hooke's Law and determine the spring constant for a rubber band from the slope of a line obtained by plotting # of washers versus displacement.

EQUIPMENT

A thick rubber band
12 heavy washers
6 inches of string
Meter stick

PROCEDURE

1. Using a scissor, carefully and safely cut a rubber band so that it becomes a single length of rubber and not a band.
2. Tie a string to one end of the rubber band.
3. Hold the rubber band vertically with the string end down and measure the length of the rubber band (not including the string).
4. Repeat your measurement 3 times.
5. Calculate the standard deviation of the length.
6. Create a data table with two columns. Column one should be labeled # of washers and column two should be labeled Displacement (m).
7. Tie two washers to the string and measure the new length of the rubber band.
8. Repeat #7, two washers at a time, until all 12 washers are used. Enter your data in the data table.

DATA ANALYSIS

1. Plot the graph of the # of Washers versus Displacement in excel. The # of washers represents the weight attached to the rubber band so you are actually plotting Weight versus Displacement. .
2. Draw the line-of-best-fit for your data. This is the line that best fits your data. Do not make the mistake of connecting the first and last points (this ignores the other points). The line-of-best-fit need not pass through any of the data points.
3. Find the slope of the line-of-best-fit.
4. What does the slope of the line-of-best-fit for # of washers versus displacement tell you about the rubber band? Since the slope of any line on a graph has units of the vertical axis divided by the horizontal axis (slope is defined as a ratio of the change in the vertical axis divided by the change in the horizontal axis), the slope of the line-of-best fit tells you the # of washers per meter of displacement for the rubber band. Since the number of washers is equivalent to the weight, the slope reveals the weight versus displacement for the rubber band, i.e., the spring constant, which is defined as force (e.g., weight) versus displacement.

Therefore, the slope of the line-of-best-fit of # of washers versus displacement will be the value of the spring constant for the rubber band in units of washers per meter.

5. Find the theoretical spring constant in the internet.
6. Calculate the percent error of your experimental result.

